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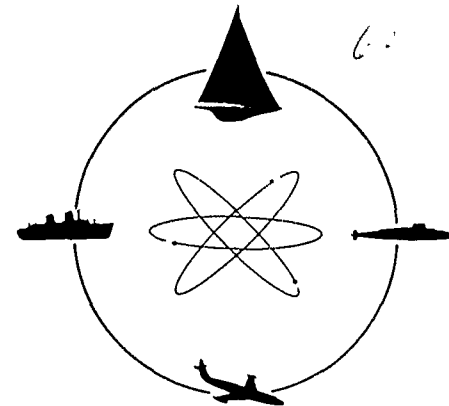
ON THE LATERAL DAMPING COEFFICIENTS
OF SUBMERGED SLENDER BODIES OF REVOLUTION

BY

Pung Nien Hu and Paul Kaplan

FEBRUARY 1962

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ABSTRACT

Expressions of the sway and yaw damping coefficients are obtained
for submerged slender bodies of revolution moving at a constant forward
speed near a free surface, derived from the Lagally theorem for unsteady
flow as extended by Cummins. The singularity distributions which generate
the flow consist of a basic doublet distribution identical with that for
a deeply submerged body and a higher order distribution to account for
the effect of the free surface. Numerical values were obtained for a
spheroid and the results were compared with the strip theory as well as
the case of zero forward speed.

NOMENCLATURE

A	amplitude of radiated waves
a_1, a_2	semi-axes of elliptical cylinder
C_1, C_2	integrals given by equations (24) and (25)
D	maximum diameter of body
E	energy of radiation
$\bar{E}$	mean energy of radiation
$\bar{F}$	force vector
F	Froude number $= U/(gL)^{1/2}$
f	submergence of body
G	Green's function
G_x	potential of a longitudinal doublet, see equation (6)
G_y	potential of a lateral doublet, see equation (10).
g	gravitational acceleration
I_1, I_2	integrals given by equations (48) and (49)
I_1'	integral given by equation (A5)
$j_n(z)$	spherical Bessel function
$K_o = g/U^2$	
$K_1 = \omega^2/g$	
k_1, k_2, k'	added mass coefficients
L	length of body
$l = L/2$	
$\bar{M} = (M_x, M_y, M_z)$	doublet strength
$\bar{N}$	moment vector
N_z	yaw damping moment
N_z'	yaw damping coefficient

$\overline{n}$, n	inward normal of surface
P_1, P_2	see equation (9)
$\overline{q} = (u, v, w)$	induced velocity by external flow
$R(x)$	radius of body
r	yaw velocity
$\overline{r}$	position vector
$S(x)$	cross-sectional area
S_o	maximum cross-sectional area
t	time
U	forward speed
v_o	sway velocity
$W = \omega(L/g)^{1/2}$	
x, y, z	cartesian coordinates
Y	sway damping force
Y'	sway damping coefficient
β	see equation (A4)
λ	see equation (9a)
ρ	density of fluid
ϕ	velocity potential
ω	frequency
$[\ ]_c$	terms containing $\cos \omega t$ in $[\ ]$

superscript indicates the order of small quantity

INTRODUCTION

In 1958, the Davidson Laboratory of the Stevens Institute of Technology, under the sponsorship of the Bureau of Ships Fundamental Hydromechanics Research Program, initiated a project to study the lateral motions of surface ships with three degrees of freedom in oblique waves. At that time, the three-dimensional effect and the forward speed effect on the damping coefficients had been detected both theoretically and experimentally by Haskind [1], Hanaoka [2], Havelock [3, 4] and Golovato [5, 6] for ships in heave and pitch motions. Since similar investigation on the lateral damping coefficients was then lacking, the present work was undertaken in order that proper correction of damping coefficients can be made in the study of lateral motions of surface ships.

The project advanced very satisfactorily. By the early summer of 1959, all expressions of the sway and yaw damping coefficients of submerged spheroids were obtained, being ready for numerical computation. Unfortunately, difficulties were encountered in the computer programming and the work was therefore temporarily suspended as both authors departed from the Davidson Laboratory in the fall of 1959. Resumption of the study was not made until the summer of 1960 when one of the authors returned to the Laboratory.

Having overcome the difficulties which arose in the computer program, the authors later learned of J. N. Newman's recent work on the damping coefficients of a submerged ellipsoid [7]. Although Newman has treated a more general problem, the present study, being based on a different approach, is by itself interesting.

In the evaluation of damping coefficients of oscillating ships,

two methods are usually employed, being based on the consideration of
the wave energy radiated from the body, or on the direct calculation
of the force and moment on the body.

The energy method was developed by Havelock [3, 8, 9] for two-
dimensional damping coefficients as well as the three-dimensional case
of zero forward speed. Generalization of the method to the case of a
moving ship was made by Newman [10, 11]. The advantage of this method
is that the damping of a submerged oscillating body can be evaluated
from the energy carried away by waves generated by the singularity
distribution which is determined by the motion of the body in an infi-
nite fluid. The correction of the singularity distribution due to the
effect of free surface can be neglected.

Haskind [1], Havelock [4] and Newman [12] evaluated the heave
and pitching damping coefficients of thin ships by integrating the pressure
over the body. Kaplan and Hu [13] calculated the damping force and
moment of submerged spheroids at zero forward speed by utilizing the
Lagally theorem extended by Cummins [14] for unsteady flows. Although
it is seen in Reference 13 that a higher-order solution of singularity
distribution is necessary for the evaluation of the damping from the
force calculation, the final expression does not show any difference
from the result obtained by energy method. Furthermore, in the study
of the motion of the body, one has to determine not only the damping
force and moment on the body but also the force and moment due to the
inertia effect. Therefore, it seems more convenient and systematic
to evaluate all forces and moments from the same approach.

In the present report, the damping coefficients were evaluated
by applying the extended Lagally theorem [14] which enables one to

determine the force and moment on the body from the singularity distribution instead of the pressure integration over the surface. This offers the advantage of reducing surface integrals to line integrals in the present study of slender bodies.

Numerical calculation of the damping coefficients of submerged prolate spheroid were carried for various speeds and frequency parameters. The results were compared with the case of zero forward speed as well as that obtained from the strip theory.

SINGULARITY DISTRIBUTIONS

The origin of the cartesian coordinate system moving with the body is taken on the undisturbed free surface with the x-y plane horizontal and the z-axis vertically upward. The slender body of revolution, fully submerged at a depth of f below the free surface, is moving forward along the x-direction at a constant speed U and oscillating at a frequency ω in sway or yaw, its center being on the negative z-axis.

The velocity potential ϕ of the fluid satisfies the Laplace equation

$$\Delta\phi = 0 , \tag{1}$$

the boundary condition on the free surface (z = 0)

$$\frac{\partial^2\phi}{\partial t^2} - 2U \frac{\partial^2\phi}{\partial t \partial x} + U^2 \frac{\partial^2\phi}{\partial x^2} + g \frac{\partial\phi}{\partial z} = 0 , \tag{2}$$

and the boundary condition on the body surface

$$\frac{\partial\phi}{\partial n} = 0 , \tag{3}$$

where n is the normal of the surface.

It is well known that the potential, for slender bodies of revolution moving in an infinite fluid, may be represented by doublet distributions on the axis of the body. The potential due to the forward motion of the body may be represented by a doublet distribution in the x-direction

$$M_x^{(1)} (x) = \frac{1 + k_1}{4\pi} S(x)U , \tag{4}$$

while the lateral oscillation of the body may be represented by a periodic doublet distribution $M_y^{(2)}(x) \cos \omega t$ in the y-direction where

$$
M_y^{(2)}(x) = \begin{cases} \dfrac{1 + k_2}{4\pi} S(x) \, v_o & \text{for sway,} \\[3em] \dfrac{1 + k'}{4\pi} S(x) \, rx & \text{for yaw.} \end{cases} \tag{5}
$$

In the above expressions, v_o and r are the amplitudes of sway and yaw velocities, respectively, being assumed to be small as compared with the forward speed; $S(x)$ is the cross-sectional area of the body, being assumed to be small as compared with the square of the body length, the superscript therefore indicates the order of the small quantity; the subscript of doublet distribution represents the orientation and k_1, k_2 and k' are the added mass coefficients for the corresponding modes of the motion, depending on the slenderness of the body.

In the case of bodies moving under a free surface, waves are generated and potential must also satisfy the boundary conditions (2) on the free surface. Havelock [15] has found that the potential of a constant doublet in the x-direction with unit strength at a point $(h, o, -f)$ below the free surface is given by

$$
G_x(x,y,z;h,o,-f) = \left(\frac{1}{r_1^3} - \frac{1}{r_2^3}\right)(x - h)
$$

$$
- \frac{4k_o}{\pi} \int_0^{\frac{\pi}{2}} \int_0^{\infty *} \frac{k \, e^{k(z-f)} \sin\left[k(x-h)\cos\theta\right]\cos(ky\sin\theta)\sec\theta \, dk \, d\theta}{k - k_o \sec^2\theta}
$$

$$+ 4K_o^2 \int_o^{\frac{\pi}{2}} e^{K_o(z-f)\sec^2\theta} \cos[K_o(x-h)\sec\theta] \cos(K_o y \sin\theta \sec^2\theta)\sec^3\theta d\theta$$

$$(6)$$

where $\quad r_1^2 = (x-h)^2 + y^2 + (z+f)^2$,

$$r_2^2 = (x-h)^2 + y^2 + (z-f)^2 , \qquad\qquad (7)$$

$$K_o = g/U^2$$

and the asterisk attached to the upper limit of integral indicates the principal part.

The potential of a periodic doublet with an amplitude of unit strength in the y-direction below a free surface may be found from the Green's function satisfying the boundary condition (2) on the free surface since a Green's function represents the potential of a source and the potential of a doublet can be obtained by differentiating that of a source with respect to the position coordinate.

Green's function satisfying the boundary condition (2) on the free surface as obtained by Peters and Stokers [16] is expressed as

$$G(x,y,z,;\xi,\eta,\zeta,t) = \frac{\cos\omega t}{[(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2]^{1/2}} - \frac{\cos\omega t}{[(x-\xi)^2 + (y-\eta)^2 + (z+\zeta)^2]^{1/2}}$$

$$+ \frac{2K_o}{\pi} \int_o^\pi \int_o^{\infty*} (\frac{P_1}{P_1 - P_2} \cdot \frac{1}{P - P_1} - \frac{P_2}{P_1 - P_2} \cdot \frac{1}{P - P_2}) e^{P(z + \zeta)}$$

$$\cdot \cos[p(x-\xi)\cos\theta - \omega t] \cos[p(y-\eta)\sin\theta]\sec^2\theta \, dp \, d\theta$$

$$-2K_o \int_\lambda^{\frac{\pi}{2}} \frac{p_1}{p_1 - p_2} e^{p_1(z+\zeta)} \sin [p_1(x-\xi) \cos \theta - \omega t] \cos [p_1(y-\eta)\sin \theta]$$
$$\cdot \sec^2 \theta \, d\theta$$

$$-2K_o \int_\lambda^{\pi} \frac{p_2}{p_1 - p_2} e^{p_2(z+\zeta)} \sin [p_2(x-\xi) \cos \theta - \omega t] \cos [p_2(y-\eta)\sin \theta]$$
$$\cdot \sec^2 \theta \, d\theta$$

$$+2K_o \int_{\frac{\pi}{2}}^{\pi} \frac{p_1}{p_1 - p_2} e^{p_1(z+\zeta)} \sin [p_1(x-\xi) \cos \theta - \omega t] \cos [p_1(y-\eta)\sin \theta]$$
$$\cdot \sec^2 \theta \, d\theta \tag{8}$$

where

$$\left.\begin{array}{c} p_1 \\ \\ \\ p_2 \end{array}\right\} = \frac{1}{4} k_o \sec^2 \theta \, [1 \overset{+}{-} (1 - \frac{4\omega U}{g} \cos \theta)^{1/2}]^2 \tag{9}$$

and

$$\lambda = \begin{cases} 0 & \text{for } 4\omega U/g \leq 1 \\ \\ \text{Arc cos } \frac{g}{4\omega U} & \text{for } 4\omega U/g > 1 \end{cases} \tag{9a}$$

Equation (8) represents the potential of a pulsating source moving at a constant forward speed under a free surface. The potential of a pulsating doublet in the y-direction can be found by differentiating equation (8) with respect to η and is expressed as

$$G_y(x,y,z;h,o,-f;t) = (\frac{1}{r_1^3} - \frac{1}{r_2^3}) \, y \cos \omega t$$

$$+ \frac{2K_o}{\pi} \int_o^\pi \int_o^{\infty^*} (\frac{p_1}{p_1 - p_2} \cdot \frac{1}{p - p_1} - \frac{p_2}{p_1 - p_2} \cdot \frac{1}{p - p_2}) \, p e^{p(z-f)}$$

$$\cdot \; \cos [p(x-h) \cos \theta - \omega t] \sin (py \sin \theta) \sec^2 \theta \sin \theta \; dp d\theta$$

$$-2K_o \int_\lambda^{\frac{\pi}{2}} \frac{p_1^2}{p_1 - p_2} e^{p_1 (z-f)} \sin [p_1(x-h) \cos \theta - \omega t] \sin (p_1 y \sin \theta)$$
$$\cdot \sec^2 \theta \sin \theta \; d\theta$$

$$-2K_o \int_\lambda^{\pi} \frac{p_2^2}{p_1 - p_2} e^{p_2 (z-f)} \sin [p_2(x-h) \cos \theta - \omega t] \sin (p_2 y \sin\theta)$$
$$\cdot \sec^2 \theta \sin \theta \; d\theta$$

$$+2K_o \int_{\frac{\pi}{2}}^{\pi} \frac{p_1^2}{p_1 - p_2} e^{p_1 (z-f)} \sin [p_1(x-h) \cos \theta - \omega t] \sin (p_1 y \sin \theta)$$
$$\cdot \sec^2 \theta \sin \theta \; d\theta \tag{10}$$

the first term containing $1/r_1^3$ in both equations (6) and (10) represents the potential of a doublet in an infinite fluid while the remaining terms represent the potential due to the image system and waves. Because of the velocities induced on the body by these remaining terms, the boundary condition (3) is infringed, and additional doublet distributions are therefore needed.

The velocity induced on the body axis by the image system and waves of the doublet distribution $M_x^{(1)}$ may be expressed as

$$(u^{(2)}, v^{(2)}, w^{(2)}) = - \nabla \phi^{(2)} \Big|_{y=o, \; z=-f} \tag{11}$$

where

$$\phi^{(2)} = \int_{-l}^{l} M_x^{(1)}(h) \left(G_x - \frac{x-h}{r_1^3} \right) dh \tag{12}$$

represents the potential due to the image system and waves of $M_x^{(1)}$, l

being the half-length of the body. In the above expression, it has been assumed that the submergence of the body is sufficiently large so that the potential due to the image system and waves is at least one order higher than that due to the doublet itself.

Equation (6) shows that G_x is an even function of y, $v^{(2)}$ therefore vanishes and the additional doublet distribution on the body axis to correct the boundary condition distorted by $\phi^{(2)}$ may be obtained as

$$\overline{M}^{(3)} = (M_x^{(3)}, o, M_z^{(3)})$$

$$= - \frac{S(x)}{4\pi} [(1 + k_1) u^{(2)}, o, (1 + k_2) w^{(2)}]. \qquad (13)$$

The image system and waves of the doublet distribution $M_y^{(2)}$ induces a velocity

$$(u^{(3)}, v^{(3)}, w^{(3)}) = - \nabla\phi^{(3)} \Big|_{y=o, z=-f} \qquad (14)$$

on the body axis, where

$$\phi^{(3)} = \int_{-l}^{l} M_y^{(2)} (G_y - \frac{y}{r_1^3} \cos \omega t) \, dh \qquad (15)$$

represents the potential due to the image system and waves of $M_y^{(2)}$. An additional doublet distribution

$$\overline{M}^{(4)} = (M_x^{(4)}, M_y^{(4)}, M_z^{(4)})$$

$$= - \frac{S(x)}{4\pi} [(1 + k_1) u^{(3)}, (1 + k_2) v^{(3)}, (1 + k_2) w^{(3)}]$$

$$\qquad (16)$$

on the body axis is therefore needed.

Neglecting the higher terms, the doublet distribution on the body axis which represents the body to the fourth order may be written as

$$\overline{M} = (M_x^{(1)} + M_x^{(3)} + M_x^{(4)}, \; M_y^{(2)} \cos \omega t + M_y^{(4)}, \; M_z^{(3)} + M_z^{(4)}).$$

$$(17)$$

The corresponding velocity potential may be written as

$$\phi = Ux + v^{(1)} \, y \cos \omega t + \int_{-\ell}^{\ell} (M_x^{(1)} + M_x^{(3)}) \, G_x dh + \int_{-\ell}^{\ell} M_y^{(2)} \, G_y dh$$

$$+ \int_{-\ell}^{\ell} M_y^{(4)} \, \frac{y}{r_1^{\,3}} \, dh + \int_{-\ell}^{\ell} M_z^{(3)} \, G_z \, dh + \int_{-\ell}^{\ell} M_x^{(4)} \, \frac{x-h}{r_1^{\,3}} \, dh$$

$$+ \int_{-\ell}^{\ell} M_z^{(4)} \, \frac{z+f}{r_1^{\,3}} \, dh,$$

$$(18)$$

where

$$v^{(1)} = \begin{cases} v_o & \text{for sway} \\ \\ rx & \text{for yaw} \end{cases}$$

$$(19)$$

and G_z represents the potential of a constant doublet in the z-direction with unit strength at a point $(h, o, -f)$ below the free surface. In the present study, only the lateral damping force and moment are of interest, it will be seen later that the doublet distribution in the z-direction makes no contribution to the lateral force and moment and the explicit expression of G_z is therefore unnecessary.

SWAY DAMPING FORCE

The Lagally theorem, which allows one to determine the force and moment on a body without carrying out the pressure integration over the body surface, has been extended by Cummins [14] to unsteady flows. The extended theorem has been applied successfully to the evaluation of stripwise damping coefficients for heave and pitch of a spheroid at zero forward speed [13] and will be therefore employed in the present study.

The extended Lagally theorem [14] for the force may be expressed as

$$\overline{F} = - 4\pi\rho\,\overline{M} \cdot \nabla\,\overline{q} - 4\pi\rho\,\frac{d\overline{M}}{dt}\,, \tag{20}$$

where $\overline{F}$ is the force acting on the doublet $\overline{M}$ and $\overline{q}$ is the velocity induced by the external flow on the doublet. In evaluating the force on a body generated by doublet distributions, the mutual interaction only contributes internal forces which will nullify each other, therefore the velocity induced by other doublets in the body should be omitted.

Now, the velocity induced by the expernal flow may be expressed as

$$\overline{q} = (-U,\ v^{(1)},\ o) - \nabla\,\phi^{(2)} - \nabla\,\phi^{(3)} + \ldots \tag{21}$$

where $v^{(1)}$, $\phi^{(2)}$ and $\phi^{(3)}$ are given by equations (19), (12) and (15), respectively, and the ellipsis dots . . . represent higher order terms of velocities induced by the image system and waves of the doublet distribution $\overline{M}^{(4)}$.

In the evaluation of the damping force, terms containing $\cos\omega t$ are of interest. Substituting equations (17) and (21) into equation (20) and keeping terms up to the fourth order, one obtains the stripwise sway damping force as

$$\frac{dY}{dx} = 4\pi\rho \; M_x^{(1)} \; [\frac{\partial^2 \phi^{(3)}}{\partial x \partial y}]_c + 4\pi\rho \cos \omega t \; \frac{\partial^2 \phi^{(2)}}{\partial y^2} - 4\pi\rho \; [\frac{dM_y^{(4)}}{dt}]_c$$

(22)

where derivatives of potentials are evaluated on the axis of body and the symbol $[\;]_c$ indicates that only the terms containing $\cos \omega t$ in $[\;]$ are to be taken. The total damping force can be obtained by integrating the above expression over the length of body. Assuming that the body is symmetric about its midship section, i.e., the cross-sectional area $S(x)$ is an even function of x, one finds that the sway damping force is

$$Y = - \frac{K_o}{2\pi} (1 + k_1)(1 + k_2) \rho U \; v_o \cos \omega t \int_{-l}^{l} S(x) \int_{-l}^{l} S(h)$$

$$\cdot \; C_1(x,h; \; 1, \; 1, \; 4) \; dh \; dx$$

$$- \frac{K_o^4}{\pi} (1 + k_1)(1 + k_2) \rho U \; v_o \cos \omega t \int_{-l}^{l} S(x) \int_{-l}^{l} S(h)$$

$$\cdot \; C_2(x,h; \; 7, \; 2) \; dh \; dx$$

$$- \frac{K_o}{2\pi} (1 + k_2)^2 \rho \; v_o \; \omega \cos \omega t \int_{-l}^{l} S(x) \int_{-l}^{l} S(h)$$

$$\cdot \; C_1(x,h; \; 0, \; 2, \; 3) \; dh \; dx \; .$$

(23)

where

$$C_1(x,h; \; m, \; n, \; q) = \sum_{i=1}^{2} \; [\int_{\lambda}^{\frac{\pi}{2}} + (-)^i \int_{\frac{\pi}{2}}^{\pi}] \; \sin^m \theta \; \tan^n \theta \; e^{-2p_1 f}$$

$$\cdot \; \frac{p_1^q}{p_1 - p_2} \cos \; [p_1 \; (x-h) \cos \theta] \; d\theta,$$

(24)

and

$$C_2(x, h; m, n) = \int_0^{\frac{\pi}{2}} \sec^m \theta \, \sin^n \theta \, e^{-2K_o \sec^2 \theta} \cos [K_o(x-h)\sec \theta] \, d\theta.$$

(25)

YAW DAMPING MOMENT

The extended Lagally theorem for the moment is expressed as [14]

$$\overline{N} = -4\pi\,\overline{\rho r}x\;(\overline{M}\cdot\nabla)\;\overline{q} + 4\pi\rho\overline{q}x\overline{M} + \frac{d}{dt}\int \rho\overline{\phi r}x\overline{n}\;dS \qquad (26)$$

where $\overline{N}$ is the moment on the doublet $\overline{M}$ about the origin, $\overline{r}$ the position vector of the location of the doublet, $\overline{n}$ the inward normal of the surface dS and the integration in the present problem is to be carried out over the peripheral surface of the strip which contains the doublet.

The moment contributed by the first two terms in equation (24) is the "Lagally" moment while the moment contributed by the surface integral is due to the time dependence of the flow.

Substituting equations (17) and (21) into the first two terms in equation (26), one obtains a stripwise yaw damping moment, attributed to the Lagally moment, as

$$\frac{dN_{z1}}{dx} = 4\pi\rho x M_x^{(1)}\;[\frac{\partial^2\phi^{(3)}}{\partial x\partial y}]_c + 4\pi\rho x\;M_y^{(2)}\cos\omega t\;\frac{\partial^2\phi^{(2)}}{\partial y^2}$$

$$-4\pi\rho\;(U + \frac{\partial\phi^{(2)}}{\partial x})\;M_y^{(2)}\cos\omega t - 4\pi\rho U\;[M_y^{(4)}]_c$$

$$+4\pi\rho\;M_x^{(1)}\;[rx\cos\omega t + \frac{\partial\phi^{(3)}}{\partial y}]_c + 4\pi\rho\;M_x^{(3)}\;rx\cos\omega t,$$

$$(27)$$

where derivatives of potentials are evaluated on the axis of body.

The surface integral in equation (27) may be evaluated approximately for the present problem. By using the method developed by Cummins [17], one obtains a stripwise yaw damping moment

$$\frac{dN_{z2}}{dz} = - \rho x \ R(x) \ [\frac{d}{dt} \int_{o}^{2\pi} \phi \cos \theta' \ d\theta']_c \ , \tag{28}$$

where $R(x)$ is the radius of cross-section, ϕ the potential evaluated on the pheripheral surface of the strip and θ' the angle in the plane of cross-section measured from the y-axis.

In examining the expression of potential ϕ given in equation (18), it is found that

$$\phi' = \phi^{(3)} + \int_{-\ell}^{\ell} M_y^{(4)} \ \frac{y}{r_1^3} \ dh \tag{29}$$

is the only part in potential ϕ which contributes to the damping moment. On the peripheral surface of the strip, equation (27) may be written as

$$\phi' = \phi^{(3)} + \int_{-\ell}^{\ell} \frac{M_y^{(4)} \ R(h) \cos \theta'}{[(x-h)^2+R^2(x)]^{3/2}} \ dh \ . \tag{30}$$

Since the integral of the second term in equation (30) peaks sharply in the vicinity of the point $x = h$, the value of the integral is mainly determined in the range of the peak. Furthermore, the body is assumed to be slender, i.e., $R < < \ell$, the integral in equation (30) may thus be evaluated approximately in the same manner as that in solving the integral equation for the doublet distribution of an elongated body of revolution [18] and equation (30) becomes

$$\phi' = \phi^{(3)} + \frac{2}{1+k_2} M_y^{(4)} \ (x) \ R \ (x) \cos \theta' \int_{-\infty}^{\infty} \frac{dh}{[(x-h)^2+R^2(x)]^{3/2}}$$

$$= \phi^{(3)} + \frac{4}{(1+k_2)R(x)} M_y^{(4)} \ (x) \cos \theta' , \tag{31}$$

where the factor $2/(1+k_2)$ is used for taking into account of the effect of finite length of the body.

Substitution of equation (16) into equation (31) yields

$$\phi' = \phi^{(3)} - R(x)\, v^{(3)} \cos\theta' \, . \tag{32}$$

The potential $\phi^{(3)}$ varies slowly on the peripheral surface of the strip since the variations of y and z are small in comparison to the length of body. One can, therefore, expand $\phi^{(3)}$ in a Taylor series about the point $(x, o, -f)$ as Cummins did in the evaluation of the moment on a slender body under regular waves [17]. This leads to

$$\phi^{(3)}(x,\ R\cos\theta',\ R\sin\theta'\ -f) = \phi^{(3)}(x,\ o,\ -f) + R\cos\theta'\ \frac{\partial\phi^{(3)}}{\partial y}$$
$$+R\sin\theta'\ \frac{\partial\phi^{(3)}}{\partial z} \, , \tag{33}$$

where higher-order terms have been neglected and the derivatives of $\phi^{(3)}$ are evaluated at the point $(x, o, -f)$.

Combining equations (14), (32) and (33), one finds that

$$\phi' = \phi^{(3)}(x,\ o,\ -f) - 2R(x)\, v^{(3)} \cos\theta' - R(x)\, w^{(3)} \sin\theta'. \tag{34}$$

It follows that the yaw damping moment given by equation (28) may be written as

$$\frac{dN_{z2}}{dx} = 2\rho x S(x)\ \left[\frac{dv^{(3)}}{dt}\right]_c . \tag{35}$$

Summing up equations (27) and (35) and integrating the result over the length of body, one obtains the total yaw damping moment

$$N_z = \frac{K_0}{2\pi} (1+k_1)(1+k') \, \rho U r \cos \omega t \int_{-l}^{l} x S(x) \int_{-l}^{l} h S(h)$$

$$\cdot \, C_1(x,h;1,1,4) \, dh \, dx$$

$$- \frac{K_0^4}{\pi} (1+k_1)(1+k') \, \rho U r \cos pt \int_{-l}^{l} x^2 S(x) \int_{-l}^{l} S(h)$$

$$\cdot \, C_2(x,h;7,2) \, dh \, dx$$

$$+ \frac{K_0^3}{\pi} (1+k_1)(k'-k_1) \, \rho U r \cos \omega t \int_{-l}^{l} x S(x) \int_{-l}^{l} S(h)$$

$$\cdot \, S_2(x,h;4,0) \, dh \, dx$$

$$+ \frac{K_0}{2\pi} (1+k')(k_2-k_1) \, \rho U r \cos \omega t \int_{-l}^{l} S(x) \int_{-l}^{l} h S(h)$$

$$\cdot \, S_1(x,h;0,2,3) \, dh \, dx$$

$$+ \frac{K_0}{\pi} (1+k') \, \rho r \omega \cos \omega t \int_{-l}^{l} x S(x) \int_{-l}^{l} h S(h)$$

$$\cdot \, C_1(x,h;0,2,3) \, dh \, dx$$

$$(36)$$

where

$$S_1(x,h;m,n,q) = \sum_{i=1}^{2} \left[\int_{\lambda}^{\frac{\pi}{2}} +(-)^i \int_{\frac{\pi}{2}}^{\pi} \right] \sin^m \theta \tan^n \theta \; e^{-2p_i f}$$

$$\cdot \frac{p_i^q}{p_1-p_2} \sin[p_i(x-h)\cos\theta] \; d\theta,$$

$$(37)$$

$$S_2(x,h;m,n) = \int_{o}^{\frac{\pi}{2}} \sec^m \theta \sin^n \theta \; e^{-2K_o f \sec^2\theta} \sin[K_o(x-h)\sec\theta] \; d\theta,$$

$$(38)$$

and C_1 and C_2 are given by equations (24) and (25).

DAMPING COEFFICIENTS

The sway and yaw damping coefficients Y' and N_z' may be obtained from the sway damping force and the yaw damping moment by the relations

$$
\left.
\begin{aligned}
Y &= Y' \, v \cos \omega t \\[2ex]
\text{and} \\[1ex]
N_z &= N_z' \, r \cos \omega t.
\end{aligned}
\right\} \tag{39}
$$

In the case of a prolate spheroid for which

$$
S(x) = S_o \left[1 - \left(\tfrac{x}{\ell}\right)^2\right] \tag{40}
$$

where S_o is the maximum cross-sectional area of the spheroid, the integrations with respect to x and h in equations (23) and (36) can be carried out explicitly. Using the integral expression for the Bessel function

$$
J_{n+\frac{1}{2}}(z) = \frac{\left(\tfrac{z}{2}\right)^{n+\frac{1}{2}}}{n! \, \pi^{1/2}} \int_{-1}^{1} (1 - t^2)^n \, e^{izt} \, dt. \tag{41}
$$

one can easily show that

$$
\int_{-1}^{1} (1 - t^2) \, e^{izt} \, dt = 4 \, J_1(z)/z \tag{42}
$$

$$
\int_{-1}^{1} t \, (1 - t^2) \, e^{izt} \, dt = i \, 4 \, J_2(z)/z \tag{43}
$$

and

$$\int_{-1}^{1} t^2 (1 - t^2) e^{izt} dt = \frac{4 j_1(z)}{z} - \frac{16 j_2(z)}{z^2} , \tag{44}$$

where

$$j_n(z) = \left(\frac{\pi}{2z}\right)^{1/2} J_{n+\frac{1}{2}}(z) \tag{45}$$

is the spherical Bessel function.

Substituting equations (23) and (36) into equation (39) and employing the relations given by equations (42) through (44), one obtains the sway and yaw damping coefficients for a prolate spheroid

$$Y' = \frac{8 K_o}{\pi} (1+k_1)(1+k_2) \rho U S_o^2 [I_1(1,2,2,2,0) + 2 K_o I_2(3,2,2,0)]$$

$$+ \frac{8 K_o}{\pi} (1+k_2)^2 \rho \omega S_o^2 I_1(2,2,1,2,0). \tag{46}$$

and

$$N_z' = - \frac{8 K_o}{\pi} (1+k_1)(1+k') \rho U S_o^2 \ell^2 [I_1(1,2,2,0,2) + 2 K_o I_2(3,2,2,0)$$

$$- 16 K_o I_2(2,2,1,1)]$$

$$+ \frac{16 K_o}{\pi} (1+k_1)(k'-k_1) \rho U S_o^2 \ell I_2(2,0,1,1)$$

$$+ \frac{8 K_o}{\pi} (k_2-k_1)(1+k') \rho U S_o^2 \ell I_1(2,2,1,1,1)$$

$$+ \frac{16 K_o}{\pi} (1+k') \rho \omega S_o^2 \ell^2 I_1(2,2,1,0,2). \tag{47}$$

where

$$I_1(m,n,q,r,s) = \sum_{i=1}^{2} \left[\int_{\lambda}^{\frac{\pi}{2}} +(-)^i \int_{\frac{\pi}{2}}^{\pi} \right] \sec^m \theta \tan^n \theta$$

$$\cdot \frac{p_1^q}{p_1-p_2} e^{-2p_1 f} j_1^r (p_1 \ell \cos \theta) j_2^s (p_1 \ell \cos \theta) \, d\theta \tag{48}$$

and

$$I_2(m,n,r,s) = \int_0^{\frac{\pi}{2}} \sec^m \theta \tan^n \theta \, e^{-2K_o f \sec^2 \theta} j_1^r (K_o \ell \sec \theta) j_2^s (K_o \ell \sec \theta) \, d\theta \tag{49}$$

The singular point $\cos \theta = g/4\omega U$ where $p_1 = p_2$ in the expression of $I_1(m,n,q,r,s)$ can be removed by an appropriate transformation of the variable given in Appendix I. Numerical evaluation of I_1 and I_2 was then carried out on digital computers* for a spheroid with a length-diameter ratio = 8 and submerged at a depth of two-tenths of the length below the free surface. The resulting damping coefficients were plotted in Figures 1 and 2, together with their values calculated for zero forward speed as well as those obtained from the strip theory (Appendices 2 and 3).

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*I_2 was evaluated on IBM 704 by the Applied Mathematical Laboratory, David Taylor Model Basin, Washington, D. C.

DISCUSSION OF THE RESULTS AND CONCLUSIONS

The lateral damping coefficients evaluated from equations (46) and (47) for a spheroid as shown in Figures 1 and 2 do not indicate any singular behavior at the point $\omega U/g = 1/4$ which is contrary to the case of longitudinal damping coefficients [4, 7, 11]. This basic difference between the lateral modes of oscillation and the longitudinal modes of oscillation can also be seen from the Green's function given by equation (8) which represents the potential of a pulsating source moving at a forward speed under a free surface. The wave terms in equation (8) contain a denominator $p_1 - p_2$ which equals $K_0 \sec^2 \theta (1 - \cos \theta)^{1/2}$ at the point $\omega U/g = 1/4$, therefore the integrands behave like $1/\theta$ near the point $\theta = 0$ and the integrals are singular.

The potential of a doublet in the longitudinal direction (x-direction) or vertical direction (z-direction), obtained by differentiating that of a source given by equation (8) with respect to ξ or ζ , has the same singular point. However the potential of a lateral doublet (in the y-direction), obtained by differentiating equation (8) with respect to η , brings out an additional term $\sin \theta$ to the integral of the wave terms, which then does not have any singular behavior. Since the flow due to longitudinal modes of oscillation can be represented by distributions of longitudinal and vertical doublets while lateral modes of oscillation can be represented by distributions of lateral doublets, the former thus has a singular point at $\omega U/g = 1/4$ and the latter does not.

Physically, longitudinal modes of oscillation generate waves which are symmetric about the median plane of ship and the amplitude of waves becomes infinite at the critcal point $\omega U/g = 1/4$. Havelock [4] inferred that this behavior probably arises from the phenomenon of resonance. In

the case of lateral modes of oscillation, however, waves are anti-symmetric about the median plane of ship, i.e., waves generated by one side of the ship are 180° out of phase with waves generated by the other side of the ship. It is therefore believed that this anti-symmetry of waves smooths out the singularity at the critical point.

Figures 1 and 2 also show that the three-dimensional effect and the forward speed effect are rather important on the damping coefficients. In order to present the results in a more practical form, the authors have tried to obtain three-dimensional correction factors for damping coefficients at various forward speed, but have found that these correction factors, un-like the case of zero forward speed treated by Havelock [3], Vossers [19], and Kaplan and Hu [13], are highly irregular with regard to the variation of frequency parameters and it is, therefore, impossible to draw any useful conclusions.

The behavior of the damping coefficients in the present study agrees very well with that in Newman's work on the ellipsoid [7] except for low frequencies at high Froude numbers. The discrepancies may have resulted from the fact that the boundary condition (3) on the body surface is satis-fied on the mean position of the surface in the present study, but it is satisfied on the instantaneous position of the surface in Newman's work. As Newman mentioned in his paper, to satisfy the boundary condition on the instantaneous position of the surface is equivalent to including the "effect of angle-of-attack" due to forward motion and the effect due to singularity distributions located on an oscillating surface. The deviation between Newman's work and present result, therefore, shows that these effects should be taken into account for cases of low frequencies at high Froude numbers. However, the question arises whether the other "effect of angle-

of-attack" due to forward motion that generates the "lift" on the body should also be included in the study of damping coefficients. It is well known that a body placed at an angle of attack in a stream exerts a lift force which, in the idealized theory of a perfect fluid, is generated by vortices shedding away from the body. If the body is also oscillating, the unsteady lift contains a damping component resulting from the energy shedding away in the oscillatory vortex wake. Since this oscillatory wake also generates waves if the body is moving under a free surface, it is therefore important to find out whether this has any appreciable effect on the damping coefficients. One of the authors is now studying the case of a thin ship which, fortunately, can be treated by the existing lifting surface theory. The result will be presented in a later report.

ACKNOWLEDGEMENTS

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APPENDIX I

Evaluation of the Integral $I_1(m,n,q,r,s)$ in the Text

Since the integrand of $I_1(m,n,q,r,s)$ given by Equation (48) has a singularity at the point $\cos\theta = g/4\omega U$ where $p_1 - p_2$ vanishes, an appropriate transformation of the integral is, therefore, necessary in order that a mechanical quadrature can be performed.

Defining a Froude number

$$F = U/(gL)^{1/2} \tag{A1}$$

and a frequency parameter

$$W = \omega\left(\frac{L}{g}\right)^{1/2}, \tag{A2}$$

where $L = 2\ell$ is the length of body, one may then rewrite equation (9) as

$$\left.\begin{array}{c} p_1 L \\ \\ p_2 L \end{array}\right\} = \frac{1}{4F^2}(1 \pm \beta)^2 \sec^2\theta , \tag{A3}$$

where

$$\beta = (1 - 4FW \cos\theta)^{1/2}. \tag{A4}$$

Since the difference between p_1 and p_2 occurs only in the sign attached to β, it is sufficient to deal with the integral

$$I_1'(m,2,q,r,s) = \int_{\frac{\pi}{2}}^{\pi} \sec^m\theta \tan^2\theta \frac{p_1^q}{p_1-p_2}$$

$$\cdot e^{-2p_1 f} j_1^r (p_1\ell \cos\theta) j_2^s (p_1\ell \cos\theta) d\theta , \tag{A5}$$

which may be written as

$$I_1'(m,2,q,r,s) = L^{1-q} \int\limits_{\frac{\pi}{2}}^{\pi} \sec^m \theta \, \tan^2 \theta \, \frac{(1+\beta)^2}{4\beta} \, [\frac{(1+\beta)^2}{4F^2 \cos^2 \theta}]^{q-1}$$

$$\cdot e^{-\frac{(1+\beta)^2}{2F^2} \cdot \frac{f}{L} \sec^2\theta} \cdot j_1^r [(\frac{1+\beta}{F})^2 \frac{1}{8 \cos \theta}]$$

$$\cdot j_2^s [(\frac{1+\beta}{F})^2 \frac{1}{8 \cos\theta}] d\theta \qquad (A6)$$

The singular point at $\beta = 0$, i.e., at $\cos \theta = g/4\omega U$, can be removed if one chooses β as the variable of integration. It follows that

$$I_1'(m,2,q,r,s) = 2^{2m + 2q - 1} \cdot L^{1-q} \cdot F^{m+1} \cdot W^{m+2q-1} \int\limits_{1}^{(1+4FW)^{1/2}} (1-\beta)^{-2q}(1-\beta^2)^{-m}$$

$$\cdot [1 - (\frac{1-\beta^2}{4FW})^2]^{1/2} \, e^{-\frac{1}{2}(\frac{4W}{1-\beta})^2 \cdot \frac{f}{L}} \cdot j_1^r [\frac{W(1+\beta)}{2F(1-\beta)}] \, j_2^s [\frac{W(1+\beta)}{2F(1-\beta)}] \, d\beta$$

$$(A7)$$

where the integrand has no singularity in the range of integration.

Integral containing p_2^q in equation (48) can be treated analogously. Equation (48) can finally be written as

$$I_1(m,2,q,r,s) = 2^{2m+2q-1} \cdot L^{1-q} \cdot F^{m+1} \cdot W^{m+2q-1} [\int\limits_{-(1+4FW)^{1/2}}^{-\mu} + \int\limits_{\mu}^{1}$$

$$- \int\limits_{1}^{(1+4FW)^{1/2}}] \, (1-\beta)^{-2q} \, (1-\beta^2)^{-m} \, [1-(\frac{1-\beta^2}{4FW})]^{1/2}$$

$$\cdot e^{-\frac{1}{2}(\frac{4W}{1-\beta})^2 \cdot \frac{f}{L}} \cdot j_1^r [\frac{W(1+\beta)}{2F(1-\beta)}] \, j_2^s [\frac{W(1+\beta)}{2F(1-\beta)}] \, d\beta \, ,$$

$$(A8)$$

where

$$\mu = \begin{cases} 0 & \text{for} \quad 4FW > 1 \\ (1-4FW)^{1/2} & \text{for} \quad 4FW \leq 1 \end{cases} \tag{A9}$$

Using a ten-point Legendre-Gauss quadrature [19], the above integral was programmed for the digital computer IBM 650 with $f/L = 0.2$. The results were then used for the evaluation of the damping coefficients given by equations (46) and (47).

APPENDIX 2

The Case of Zero Forward Speed

If the forward speed of the oscillating spheroid is zero, the velocity potential for the motions of the fluid satisfies the boundary condition

$$\omega^2 \phi' = g \frac{\partial \phi'}{\partial z} \tag{A10}$$

on the free surface $z = 0$. The potential of a unit doublet satisfying the above boundary conditions may be obtained by differentiating the Green's function [16] with respect to the position coordinate. It follows that the potential ϕ' may be written as

$$\phi' = e^{i\omega t} \int_{-\ell}^{\ell} \left[\frac{y}{r_1^3} + \frac{y}{r_2^3} + 2K_1 \frac{y}{a'} \int_0^{\infty *} \frac{J_1(ka')}{k - K_1} e^{K_1(z-f)} k \, dk \right] M_y^{(2)}(h) \, dh$$

$$+ i \, e^{i\omega t} \, 2\pi \, K_1^2 \, y \, e^{K_1(z-f)} \int_{-\ell}^{\ell} \frac{J_1(K_1 a')}{a'} M_y^{(2)}(h) \, dh \, , \tag{A11}$$

where

$$a'^2 = (x-h)^2 + y^2 . \tag{A12}$$

$$K_1 = \omega^2/g . \tag{A13}$$

J_1 is the Bessel function of the first order and $M_y^{(2)}$ is the doublet strength given by equation (5).

At large distances from the body, terms in the brackets of equation (A11) diminish more rapidly than the last terms. The real part of the asymtotic velocity potential may be found as

$$\phi' = -2\pi K_1^2 \, e^{K_1(z-f)} \int_{-\ell}^{\ell} \left(\frac{2}{\pi K_1 a'}\right)^{1/2} \frac{y}{a'} \sin\left(\omega t - K_1 a' + \frac{3}{4}\pi\right) M_y^{(2)}(h) \, dh$$

$$\text{(A14)}$$

as $a' \to \infty$.

The energy radiated from the body may be determined by integrating the product of the pressure and the radial velocity over the surface of an infinitely long vertical cylinder of large radius, i.e., the energy of radiation may be expressed as

$$E = -\lim_{a \to \infty} \int_{-\infty}^{0} \int_{0}^{2\pi} \rho \, \frac{\partial \phi}{\partial t} \, \frac{\partial \phi}{\partial a} \, a \, d\Theta \, dz \qquad \text{(A15)}$$

where a, Θ and z are the cylindrical coordinates with a the radial axis, Θ the angle and z the vertical axis. Since

$$x = a \cos \Theta \qquad\qquad \text{and} \qquad\qquad y = a \sin \Theta ,$$

$$\text{(A16)}$$

one thus has

$$a' = [(x-h)^2 + y^2]^{1/2} = a - h \cos \Theta \qquad\qquad \text{(A17)}$$

and

$$\frac{1}{a'} \cong \frac{1}{a} \qquad\qquad \text{(A18)}$$

as $a \to \infty$. It follows that

$$\phi' = -2\pi K_1^2 \, e^{K_1(z-f)} \left(\frac{2}{\pi K_1 a}\right)^{1/2} \sin \Theta \int_{-\ell}^{\ell} \sin\left(\omega t - K_1 a - K_1 h \cos \Theta + \frac{3}{4}\pi\right) M_y^{(2)}(h) \, dh$$

$$\text{(A19)}$$

asymptotically, and

$$E = \lim_{a \to \infty} 4\pi \rho K_1^3 \, \omega \, e^{-2K_1 f} \int_0^{2\pi} \sin^2 \theta \, [\int_{-\ell}^{\ell} \cos (\omega t - K_1 a - K_1 h \cos \theta + \tfrac{3}{4}\pi) \cdot M_y^{(2)}(h) dh]^2 d\theta \tag{A20}$$

The mean energy radiation is thus found to be

$$\bar{E} = 2\pi \rho K_1^3 \, \omega \, e^{-2K_1 f} \int_0^{2\pi} (P^2 + Q^2) \sin^2 \theta \, d\theta \tag{A21}$$

where

$$P + iQ = \int_{-\ell}^{\ell} M_y^{(2)}(h) \, e^{iK_1 h \cos \theta} \, dh. \tag{A22}$$

The relation between the damping coefficient and the mean energy radiation is given by

$$\bar{E} = \begin{cases} \tfrac{1}{2} Y' \, v^2 & \text{for sway} \\[2em] \tfrac{1}{2} M_z' \, r^2 & \text{for yaw,} \end{cases} \tag{A23}$$

one finds that the damping coefficient at zero forward speed

$$Y' = \frac{1}{4\pi} \rho K_1^3 \, \omega \, (1+k_2)^2 \, e^{-2K_1 f} \int_0^{2\pi} (P_1^2 + Q_1^2) \sin^2 \theta \, d\theta \tag{A24}$$

and

$$N_z' = \frac{1}{4\pi} \rho K_1^3 \, \omega \, (1+k')^2 \, e^{-2K_1 f} \int_0^{2\pi} (P_2^2 + Q_2^2) \sin^2 \theta \, d\theta \tag{A25}$$

where

$$P_1 + iQ_1 = \int_{-\ell}^{\ell} S(h) \, e^{iK_1 h \cos \Theta} \, d\Theta \qquad (A26)$$

and

$$P_2 + iQ_2 = \int_{-\ell}^{\ell} hS(h) \, e^{iK_1 h \cos \Theta} \, d\Theta. \qquad (A27)$$

For a prolate spheroid, equations (40) and (41) may be employed and the above expressions become

$$P_1 + iQ_1 = \frac{4 \left(\frac{\pi}{2}\right)^{1/2} S_o \ell}{(K_1 \ell \cos \Theta)^{3/2}} \, J_{5/2} \, (K_1 \ell \cos \Theta) \qquad (A28)$$

and

$$P_2 + iQ_2 = i \frac{4 \left(\frac{\pi}{2}\right)^{1/2} S_o \ell^2}{(K_1 \ell \cos \Theta)^{3/2}} \, J_{5/2} \, (K_1 \ell \cos \Theta). \qquad (A29)$$

The damping coefficients given by equations (A24) and (A25) were evaluated numerically for the same spheroid treated in the text, and the results were plotted in Figures 1 and 2.

APPENDIX 3

Strip Theory

Havelock [8] has found that the wave amplitude at infinity, generated by an infinite elliptic cylinder oscillating horizontally at a depth of below the free surface, is given by

$$A = 2\pi K_1 a_1 \frac{v_o}{\omega} (\frac{a_1 + a_2}{a_1 - a_2})^{1/2} e^{-K_1 f} \cdot I_1 [K_1 (a_1^2 - a_2^2)^{1/2}].$$

(A30)

where a_1 and a_2 $(a_1 > a_2)$ are the semi-axes of the cylinder.

For a circular cylinder, the wave amplitude may be obtained by expanding the modified Bessel function I_1 in equation (A30) for small $a_1^2 - a_2^2$ and setting $a_1 = a_2 = R$ after the cancellation of $a_1 - a_2$ in the denominator. This yields

$$A = 2\pi K_1^2 R^2 \frac{v_o}{\omega} e^{-K_1 f}.$$

(A31)

The mean energy radiation per unit length of the circular cylinder is $\rho g \omega A^2 / 2 K_1$. Assuming that this holds for each strip of the spheroid, then the mean energy radiation per unit length of a strip dh is

$$dE = 2 \rho K_1^2 \omega v_o^2 e^{-2K_1 f} S^2(h) dh,$$

(A32)

for sway oscillation and

$$dE = 2 \rho K_1^2 \omega r^2 e^{-2K_1 f} h^2 S^2(h) dh$$

(A33)

for yaw oscillation. Integrating equations (A32) and (A33) over the length of the body and substituting equation (40), one obtains the damping co-

efficients of a spheroid based on strip theory

$$Y' = \frac{64}{15} \rho K_1^2 \omega e^{-2K_1 f} S_o^2 \ell. \qquad (A34)$$

and

$$M_z' = \frac{64}{105} \rho K_1^2 \omega e^{-2K_1 f} S_o^2 \ell^3. \qquad (A35)$$

Numerical computations were made for the same spheroid treated in the text and the results were plotted in Figures 1 and 2.
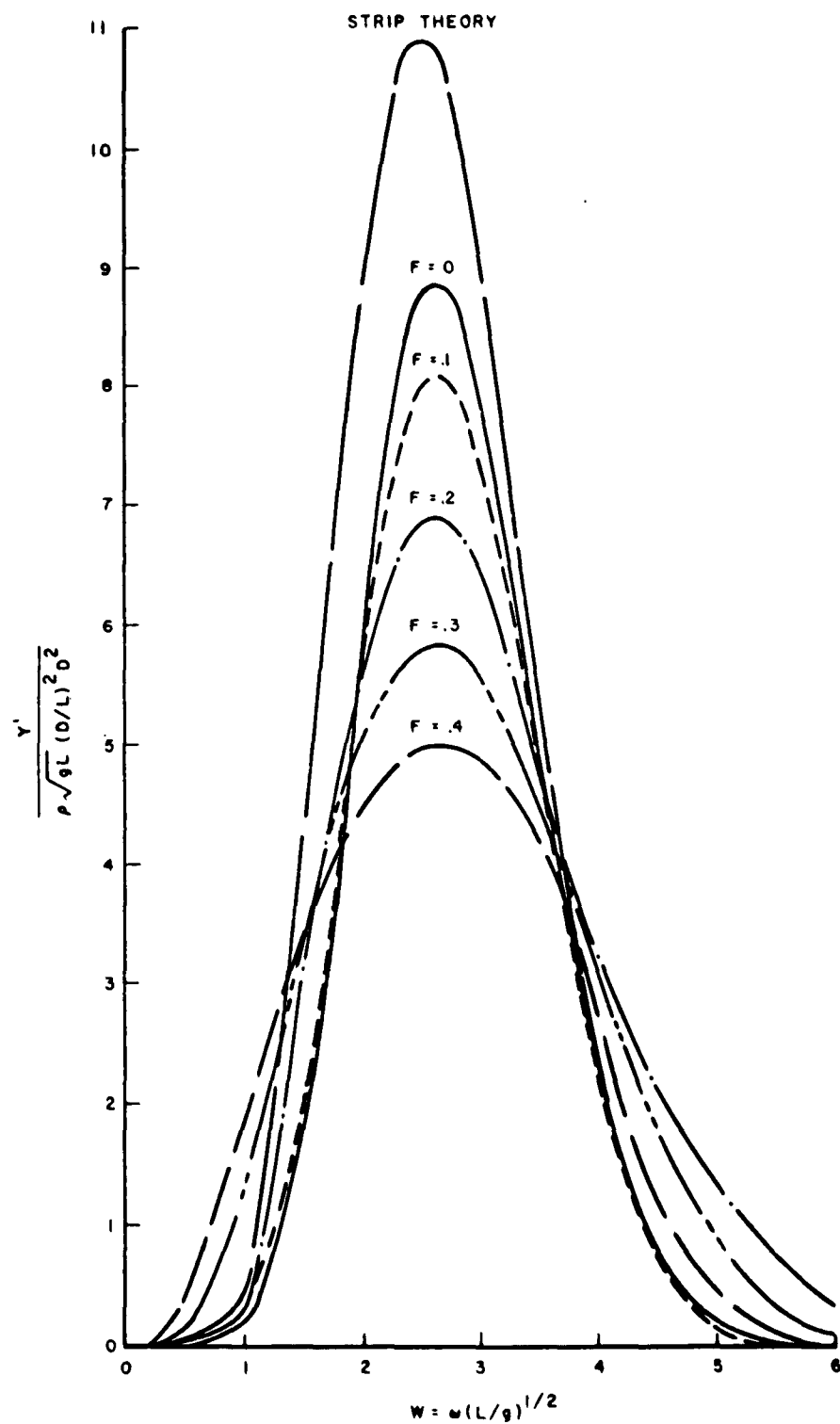
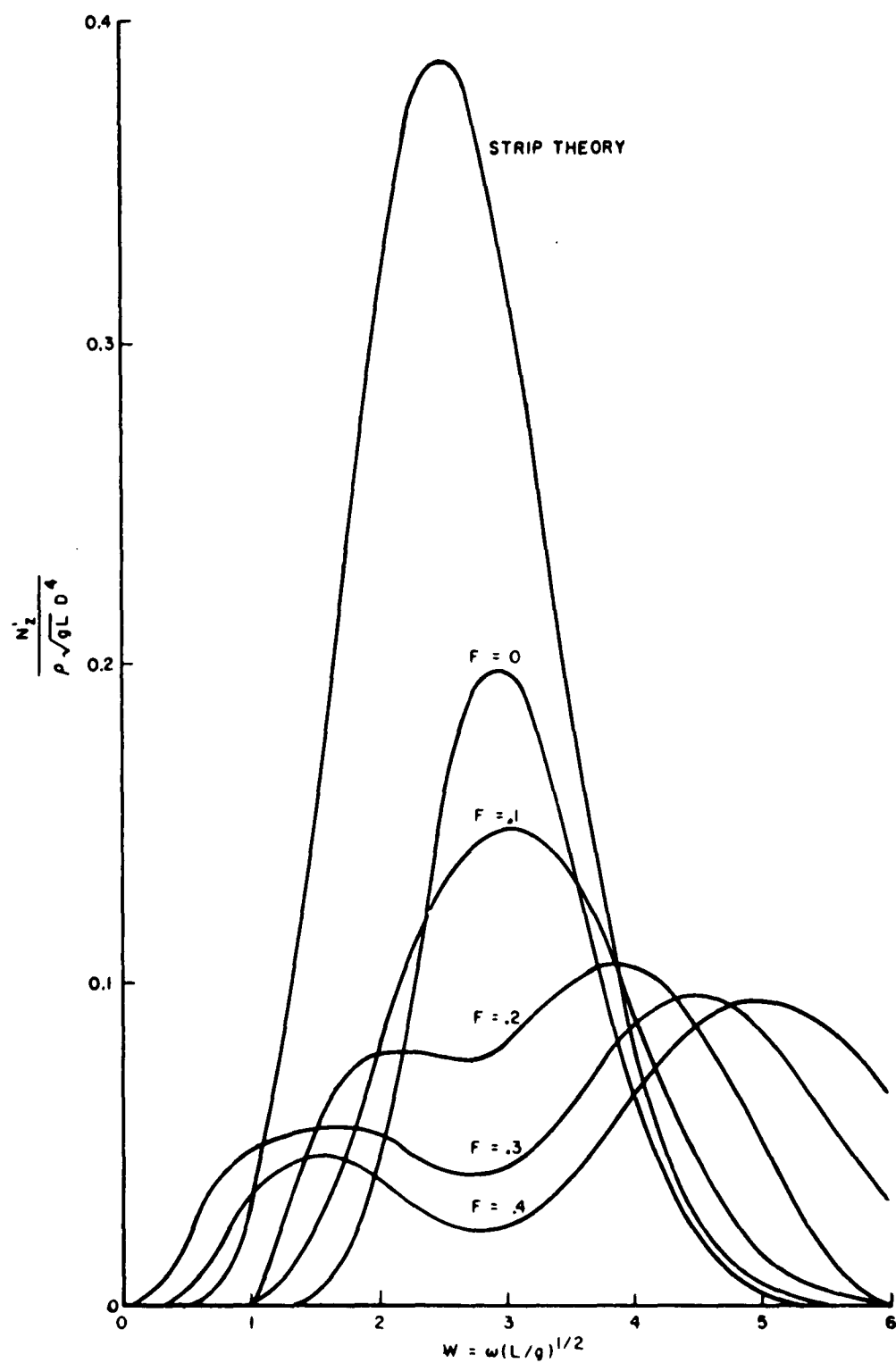FIGURE 1.   SWAY DAMPING COEFFICIENT FOR THE SPHEROID
L/D = 8, L/f = 5

FIGURE 2. YAW DAMPING COEFFICIENT FOR THE SPHEROID
L/D = 8, L/f = 5

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ON THE LATERAL DAMPING COEFFICIENTS OF SUBMERGED SLENDER BODIES

OF REVOLUTION by Pung Nien Hu and Paul Kaplan, February 1962

UNCLASSIFIED

Expressions of the sway and yaw damping coefficients are obtained for submerged bodies of revolution moving at a constant forward speed near a free surface, derived from the Lagally theorem for unsteady flow as extended by Cummins. The singularity distributions which generate the flow consist of a basic doublet distribution identical with that for a deeply submerged body and a higher order distribution to account for the effect of the free surface. Numerical values were obtained for a spheroid and the results were compared with the strip theory as well as the case of zero forward speed.